Essay About Twin Primes

The intent of this essay is not to try to prove that the twin primes are infinite. We would just like to add another way so that others interested in Number Theory can help in elucidating this mystery.

The conjecture of Polignac states that each natural even number is equal to the difference of two primes; but this conjecture, it seems, has not yet been proven. However, we note that there is a certain correlation of that thesis with the foundations of our previous study, as proposed in **"Goldbach - New Conjecture"**, which led to this monograph on twin primes.

Initially we will summarize the proposal equivalent to Goldbach's conjecture, which can be examined in detail at http://www.apex.eti.br.

All natural > 1 can be represented by the mean of two primes **p** and **q** equidistant from a natural **n**, through an integer index **k**, such that:

$\mathbf{n = (p + q) \div 2}$, being

$\mathbf{p = n - k}$ and

$\mathbf{q = n + k}$.

There is symmetry involving **n** and both primes **p** and **q** with amplitude

$3 \;\cdots\; \mathbf{n} \;\cdots\; 2 \times n - 3.$

We will use these concepts as a foundation for the study that we will present about the **twin primes**, the pairs (g, h), with

$|h - g| = 2.$

So we have, within our formulation, for a given k_g:

$g = (p_g + q_g) \div 2,$

$p_g = (g - k_g),$

$q_g = (g + k_g);$

And for a given k_h:

$h = (p_h + q_h) \div 2,$

$p_h = (h - k_h),$

$q_h = (h + k_h).$

For example, below are some symmetries with the pair (71, 73).

Pair (g, h)	k	p	q
$g = 71$ $h = 73$	$k_g = 12$	$p_g = 59$	$q_g = 83$
	$k_h = 6$	$p_h = 67$	$q_h = 79$
	$k_g = 18$	$p_g = 53$	$q_g = 89$
	$k_h = 36$	$p_h = 37$	$q_h = 109$
	$k_g = 30$	$P_g = 41$	$q_g = 101$
	$k_h = 30$	$p_h = 43$	$q_h = 103$
	$k_g = 66$	$p_g = 5$	$q_g = 137$
	$k_h = 66$	$p_h = 7$	$q_h = 139$

According to the conjecture, both symmetries exist — individually, of course — and therefore the index **k** behaves randomly, as seen in the examples with

$k_g = 12$ and $k_h = 6$ or

$k_g = 18$ and $k_h = 36$,

Without connection between **g** and **h**.

However, we observed the possibility of finding in each pair chosen for testing, many cases where index **k** could be unique, as seen in two other examples, with

$k_g = k_h = 30$ or

$k_g = k_h = 66$,

And there is a link between **g** and **h**.

This condition — $\mathbf{k = k_g = k_h}$ — is the basis of this study and we are interested only when and if it can occur; in this situation:

For any pair **(g, h)** we can do

$(g - k) = p$,

$(g + k) = q$,

$(h - k) = p + 2$,

$(h + k) = q + 2$.

Therefore we will have:

$\mathbf{g = (p + q) \div 2}$ and

$\mathbf{h = [\,(p + 2) + (q + 2)\,] \div 2.}$

Looking only for these solutions we had some success with several tests, which induced us to the theory that follows and to distinguish the twin primes in that $\mathbf{k_g \neq k_h}$ of others in that $\mathbf{k_g = k_h}$ we adopt the following concept:

Identical Twin Primes

Are those in which at least one **k**, simultaneously, satisfies a pair **(g, h)**.

Therefore, among the symmetries of the previous examples, only the following identities can be considered **identical twin primes**:

Pair (g, h)	**k**	**p**	**q**
$g = 71$ $h = 73$	30	$p_g = 41$	$q_g = 101$
		$p_h = 43$	$q_h = 103$
	66	$p_g = 5$	$q_g = 137$
		$p_h = 7$	$q_h = 139$

In iterative surveys with k, we were able to conduct symmetry in this way for identical twin primes of small magnitude, and we realized that it was possible to obtain them many times. In *Table 1* we have the result of the first pairs.

However, as we can see in the table, we have already started with two pairs where we cannot obtain simultaneous symmetry and, later, we stop at pair (197, 199), also in the same situation; that is, there are impossible cases if we require **n > k**.

At this point we will pause in our study of twin primes.

Let's revisit the original conjecture considering what would happen if we could expand the symmetry to negative values, that is, if we could make **k > n** possible.

Without restriction for **k**, one immediately observes symmetry with infinite amplitude.

Similarly, as in the initial conjecture, equalities are maintained:

$\mathbf{n = (p + q) \div 2}$, being

$\mathbf{p = n - k}$ and

$\mathbf{q = n + k}$;

Where are primes:

$\mathbf{| p |}$ **and** $\mathbf{q}$.

Note that **any** integers can now be obtained, and that, in particular:

$\mathbf{n = 0}$ with any primes, for $p + q = 0$;

$\mathbf{n = 1}$ with any pairs of twin primes;

$\mathbf{n < 0}$ it is a reflection of $\mathbf{n > 0}$.

Table 1			
Pairs	**k**	**p_g | p_h**	**q_g | q_h**
3 5	impossible		
5 7	impossible		
11 13	6	5	17
		7	19
17 19	12	5	29
		7	31
29 31	12	17	41
		19	43
41 43	30	11	71
		13	73
59 61	42	17	101
		19	103
71 73	30	41	101
		43	103
101 103	90	11	191
		13	193
107 109	90	17	197
		19	199
137 139	132	5	269
		7	271
149 151	42	107	191
		109	193
179 181	168	11	347
		13	349
191 193	90	101	281
		103	283
197 199	impossible		

The search iteration can be obtained as follows:

For n even:

$k = 1, 3, 5, \cdots \infty.$

For n odd:

$k = 2, 4, 6, \cdots \infty.$

But, let's return to our study, when we have **identical twin primes**.

The proposition assumes the bond between twin primes **g** and **h**, when and if

$\mathbf{k = k_g = k_h.}$

And, except for the pair (3, 5), we have the iteration of k boils down to:

$k = 6, 12, 18, \cdots \infty$

Until simultaneously appear the primes:

$|p|$ and q;

$|p + 2|$ and $q + 2$.

In summary, we have:

$p_g = (g - k),$

$q_g = (g + k),$

$p_h = (g - k + 2)$ and

$q_h = (g + k + 2).$

Without restriction for **k** let's see those impossible identities of our table.

Pairs	**k**	$\mathbf{p_g \mid p_h}$	$\mathbf{q_g \mid q_h}$
3	8	−5	11
5		−3	13
5	12	−7	17
7		−5	19
197	630	−433	827
199		−431	829

Interesting; it is possible to obtain symmetry.

In addition, among the set of the first 1048576 odd primes we have:

3199 identities representing the identical twin primes (5, 7);

1669 identities for (197, 199).

Curiously, even with infinite amplitude, there is only one identity for (3, 5), with $k = 8$, and it is an exercise for the reader to demonstrate the fact.

Hint: other twin primes are of the form $(6m - 1, 6m + 1)$ for some natural m and therefore, $g \equiv 2 \pmod 3$ and $h \equiv 1 \pmod 3$.

For symmetry of pairs of identical twin primes it is necessary that, in general, more than one coincidence occurs for **g** and **h** — in isolation — and such that at some point, for identical **k** values, we find equidistant primes.

For 12484 first pairs of twins primes, with the same set of primes already mentioned, we found multiple identities intended, the lowest number being 1035 for (1302017, 1302019) and the highest value was 9468 for (180179, 180181).

To illustrate: among 2188 identities for (41, 43) we selected some cases:

Pair	**k**	$p_g \mid p_h$	$q_g \mid q_h$
41 43	30	+11	71
		+13	73
	18000	−17959	18041
		−17957	18043
	1008000	−1007959	1008041
		−1007957	1008043
	2070000	−2069959	2070041
		−2069957	2070043
	2163000	−2162959	2163041
		−2162957	2163043
	3894000	−3893959	3894041
		−3893957	3894043
	4092000	−4091959	4092041
		−4091957	4092043
	5010000	−5009959	5010041
		−5009957	5010043

So it seems that being infinite amplitude, with infinite prime numbers, it is impossible to determine for each chosen pair how many representations result in identical twin primes, excluding, as already mentioned, the pair (3, 5) with a single identity.

However, one remaining question remains: can all twin primes be identified as identical? That is: are sets equivalents?

Then, reiterating, if

(g, h) are identical twin primes, we have:

$g = (p + q) \div 2,$

$h = [(p + 2) + (q + 2)] \div 2$

And as a consequence, are also twin primes the pairs:

$(p, p + 2)$ and

$(q, q + 2)$.

Therefore, under these conditions, each pair of identical twin primes leads to other twin primes, however not necessarily identical!

But by exploring the previous question:

- If we could ensure that all twin primes can also be identical

and

- If there were one last pair of identical twin primes **(g_u, h_u)**.

It would mean that the last pair of identical twin primes would forward to the another pair of identical twin primes of greater magnitude, which would be an incongruity.

Conclusion: if it were so, forcibly, the twin primes numbers would be infinite.

P.S.

One last question:

Since the twin primes are a particular case of the Conjecture of Polignac, would it be possible to expand this new hypothesis?

I believe so!

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If you can help make this text more readable, I thank you.